Optimal Control for Remote Patient Monitoring with Multidimensional Health States

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Outline

1. Background and Motivation
2. Service Architecture
3. Optimal Policy

Remote Patient Monitoring

- Technology enabled healthcare

Continuous Glucose Monitors

Smartwatches

Implantable Devices

Images created using OpenAI's DALL-E 3 model. Prahalad et al. (2024). Nature Medicine, 1-9., Masoumian Hosseini et al. (2023). BMC Medical Informatics and Decision Making, 23(1), 248. , Zinzuwadia et al. (2023). In Emerging Practices in Telehealth (pp. 97-115). Academic Press.

Remote Patient Monitoring

Ordinary

Intensive

Images created using OpenAI's DALL-E 3 model

Remote Patient Monitoring

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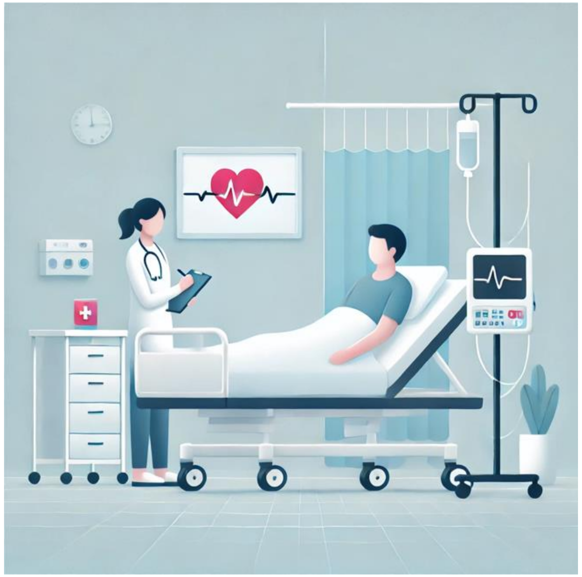
Images created using OpenAI's DALL-E 3 model

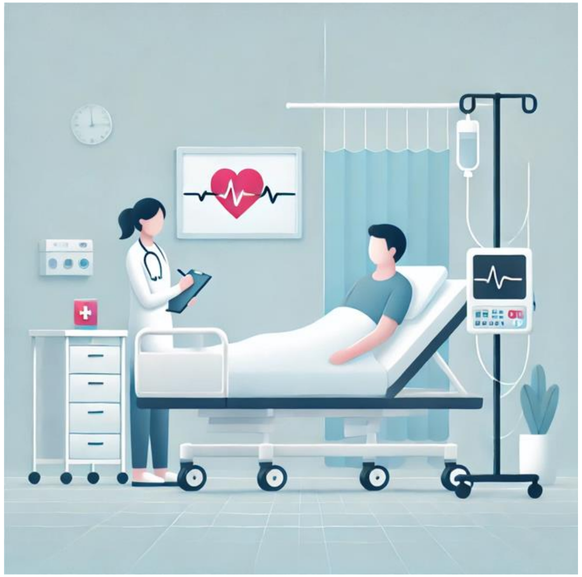
Remote Patient Monitoring



Critical State

Images created using OpenAI's DALL-E 3 model

Remote Patient Monitoring



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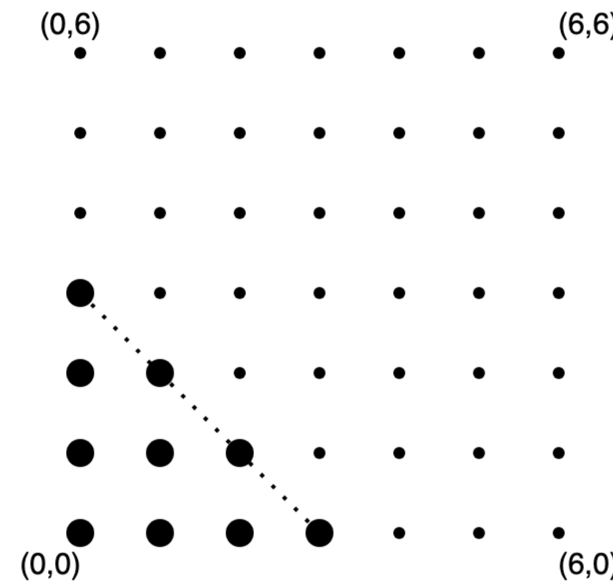
Images created using OpenAI's DALL-E 3 model

Tiered Service Architecture

Multidimensional Health State

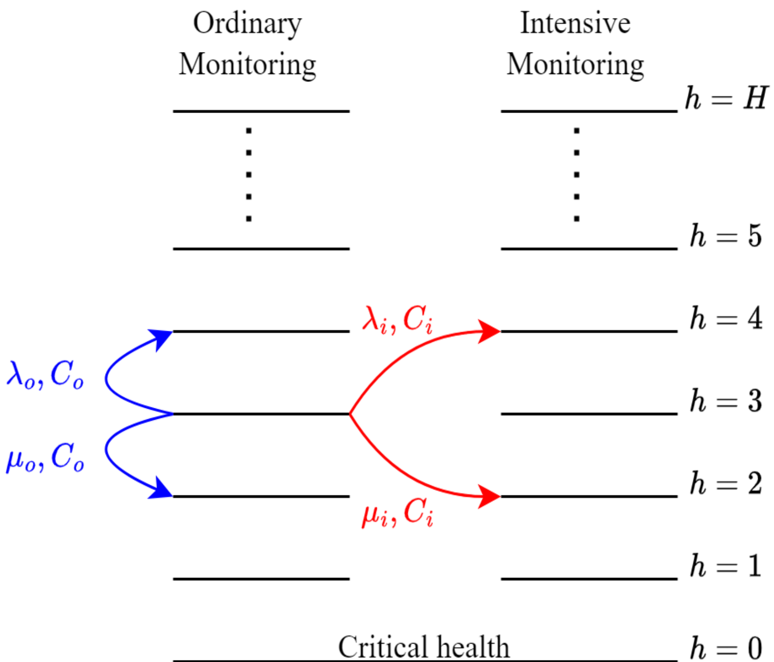
- Multidimensional health states: $\boldsymbol{h} = \left(h^{(1)}, h^{(2)}, \dots, h^{(n)}\right)$
  - $h^{(i)} \in \{0,1, \dots, H\}$
  - n different health measurements
  - "Higher" health state ⇒ Better health
- For better visualization, we work with 2 states
  - $\boldsymbol{h} = (h^{(x)}, h^{(y)})$
- Set of critical health states - $\mathcal{H}_C = \{\boldsymbol{h} \mid g(\boldsymbol{h}) \leq c\}$
  - Health falls below certain threshold
  - Examples:
    - Both measurements are zero
    - Either measurement is zero



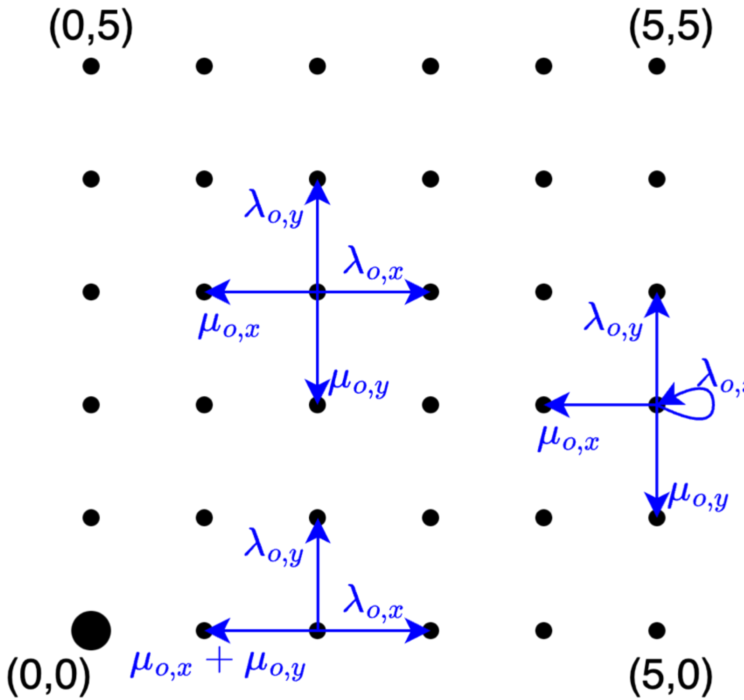
Markovian Model

- **Markov Decision Process**
- **Time period**: $t \in \{0,1,2 \ldots\}$
- **Health states**: $h_t = \left(h_t^{(x)}, h_t^{(y)}\right)$
- **Monitoring States**: $m_t \in \{o, i\}$
  - o: Ordinary Monitoring
  - i: Intensive Monitoring
- **Overall State**: $s_t = (h_t, m_t)$
- **Action/Control**: $a_t \in \{o, i\}$
- **Costs:** C_o, C_i, C_c
- Evolution stops when a critical health state is reached

One-Dimensional Model[1]



Multidimensional Model



- [1] S. Chandak, I. Thapa, N. Bambos, D. Schienker, "Tiered Service Architecture for Remote Patient Monitoring", IEEE Healthcom 2024

Assumptions

- The transition probabilities satisfy:
  - $\lambda_{i,x} \geq \lambda_{o,x}$
  - $\lambda_{i,y} \geq \lambda_{o,y}$
- The costs satisfy:
  - $0 \leq C_o \leq C_i << C_c$

Optimal Policy

Optimal Control

- Value Function

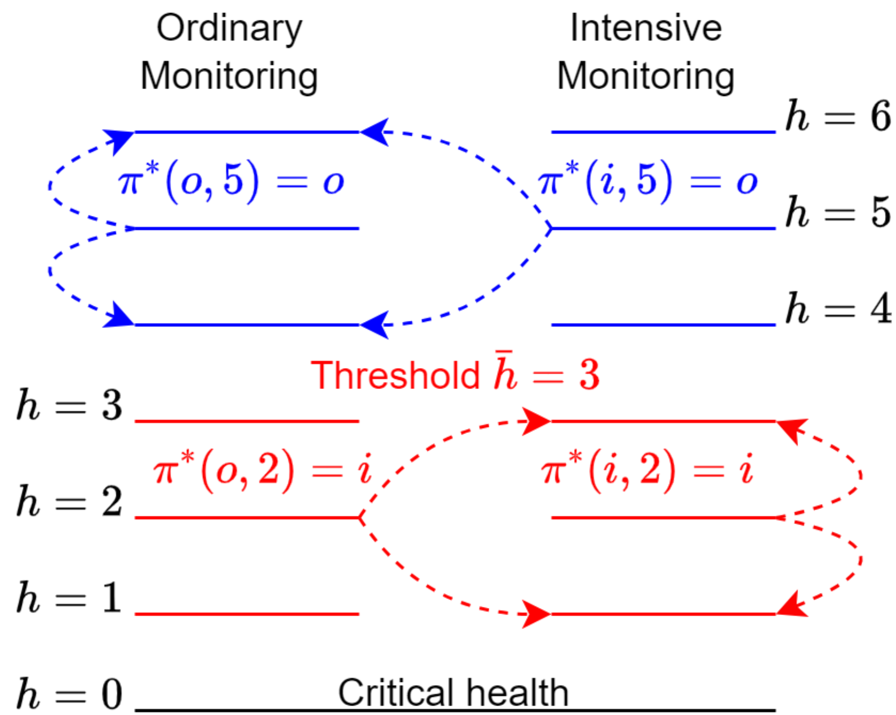
$$V^*(s) = \min_{a \in \{o,i\}} \left\{ c(s,a) + \gamma \sum_{s' \in S} P(s' \mid s,a) V^*(s') \right\}$$

- Optimal Policy

$$\pi^*(s) = \operatorname*{argmin}_{a \in \{o,i\}} \left\{ c(s,a) + \gamma \sum_{s' \in S} P(s' \mid s,a) V^*(s') \right\}$$

Intuition from One-Dimensional Model[1]

- Threshold-based policy
  - When $h > \bar{h}$:
    - Ordinary Monitoring
  - When $h \leq \bar{h}$:
    - Intensive Monitoring
- Proof Technique:
  - Moment Generating Function (MGF) of the hitting time of critical health state

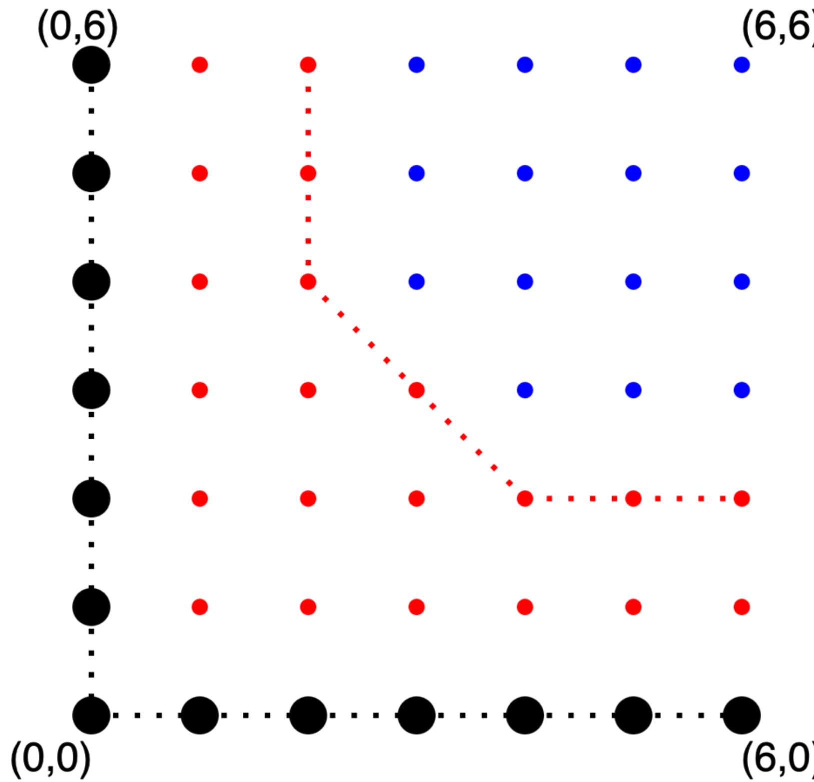


- [1] S. Chandak, I. Thapa, N. Bambos, D. Schienker, "Tiered Service Architecture for Remote Patient Monitoring", IEEE Healthcom 2024

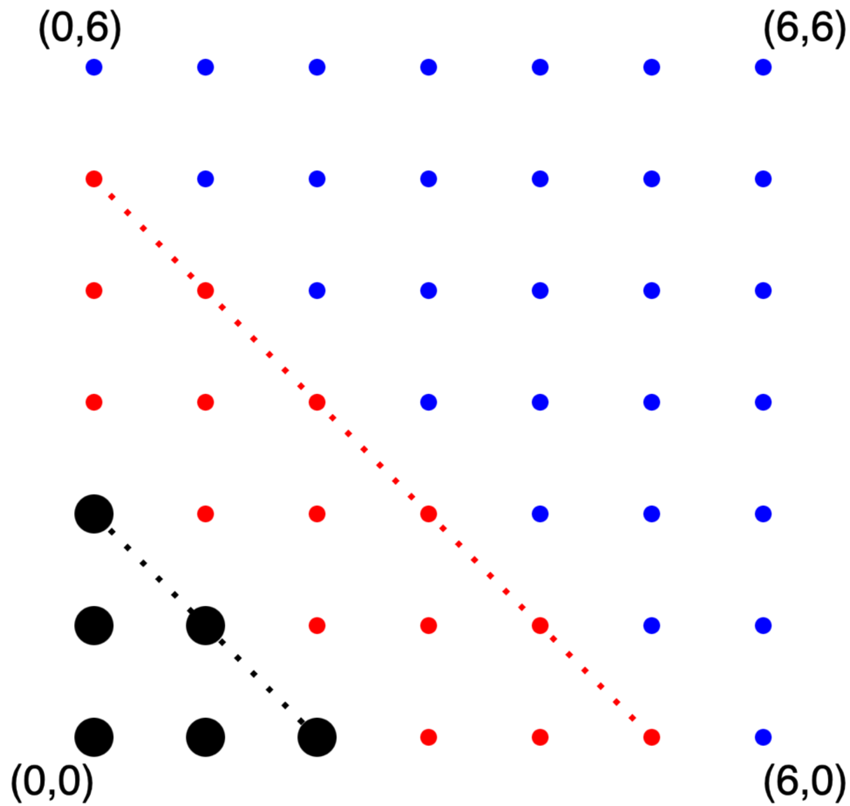
Switching Curve Policies

- Set of critical health states – $\mathcal{H}_C = \{\boldsymbol{h} \mid g(\boldsymbol{h}) \leq c\}$
- Optimal Policy characterized by some function $f(\cdot) : \mathcal{H} \mapsto \mathbb{R}$
  - If $f(\boldsymbol{h}) \leq 0$: Intensive Monitoring
  - If $f(\boldsymbol{h}) > 0$: Ordinary Monitoring
- $f(\boldsymbol{h}) = 0$ is the Switching Curve
- Same intuition as before:
  - Shape of switching curve dictated by the hitting time

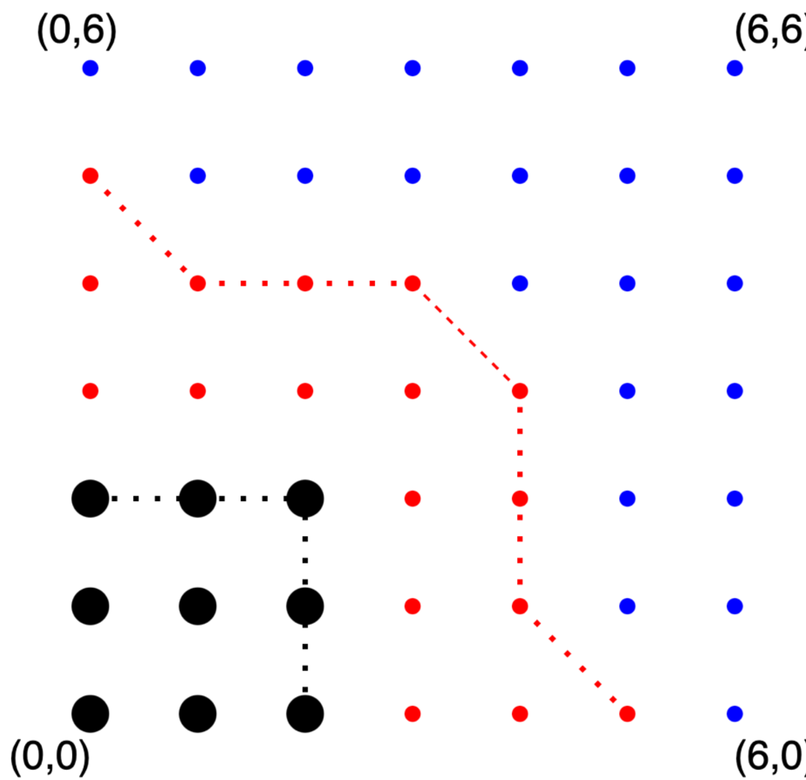
Critical Health State - 1



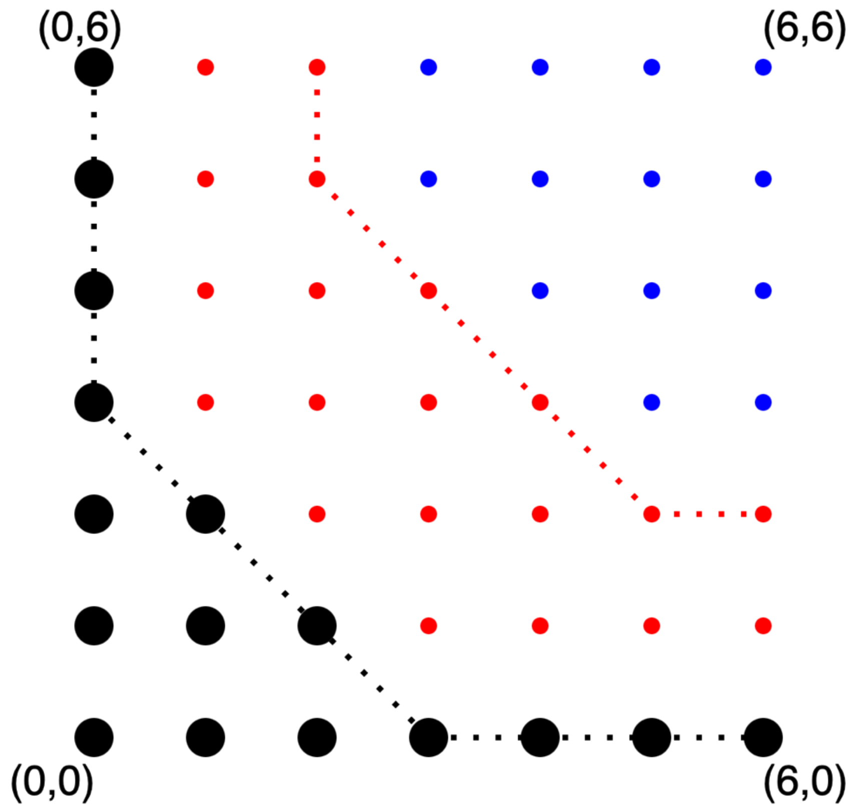
Critical Health State - 2



Critical Health State - 3



Critical Health State - 4



Conclusion

- Tiered monitoring system with multidimensional health states
- Policies based on Switching Curves
- Extensions
  - Learning
  - Non-Markovian model

Thank You

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