# Learning Desirable Equilibria for Unknown Multi-Agent Systems

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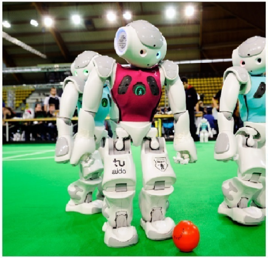
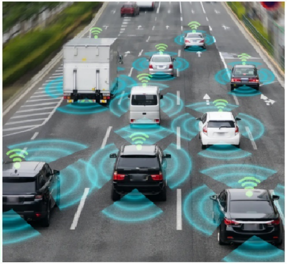
# Outline

- Overview
- Quality of Service
- Tug-of-Peace
- Summary

# Overview

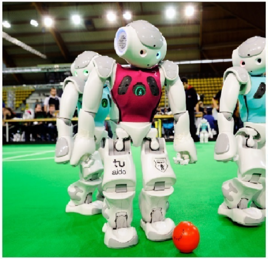
# Multi-Agent Systems

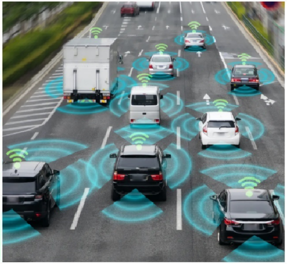




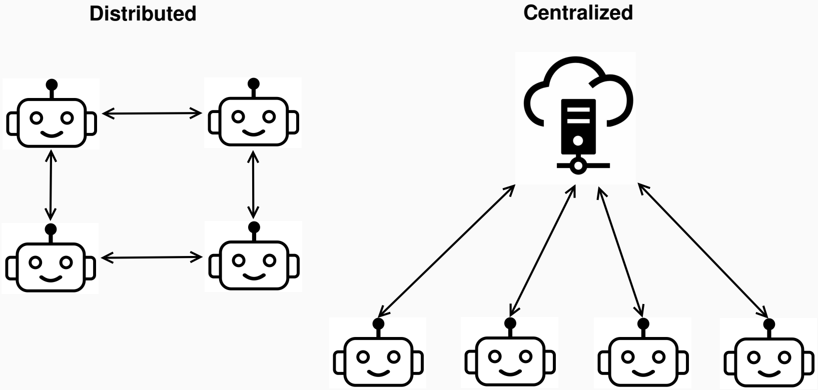
# Multi-Agent Games

- Game with $N$ agents
- Each player $n$ takes action $x_n$
- Utility (Reward): $u_n(x_1, \ldots, x_N)$

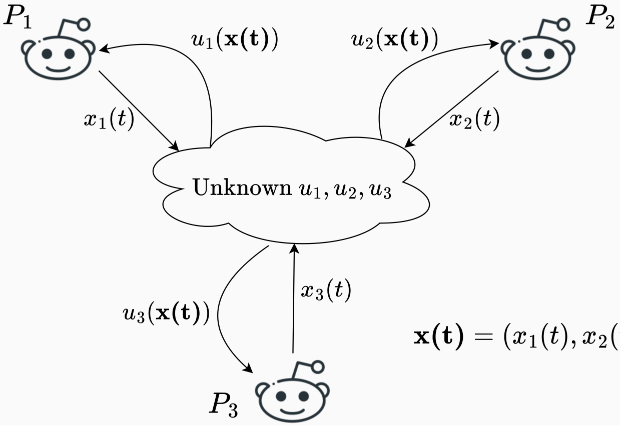




# Challenges: Distributed System

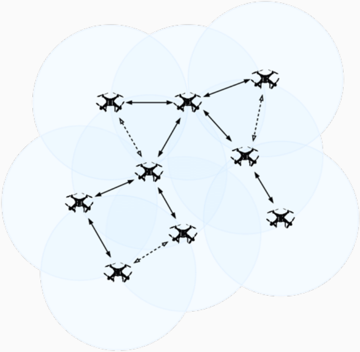


# Challenges: Bandit Feedback




$$\mathbf{x(t)} = (x_1(t), x_2(t), x_3(t))$$

# Challenges: Limited Communication



# Research in Multi-Agent Systems

- Equilibrium Bandits: Learning Optimal Equilibria of Unknown Dynamics[1]
- Tug of Peace: Distributed Learning for Quality of Service Guarantees[2]

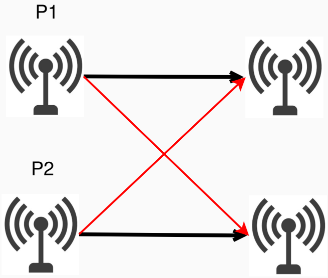
[1] Chandak, Bistritz, Bambos: in *International Conference on Autonomous Agents and Multiagent Systems (AAMAS) 2023*
[2] CBB: submitted to *IEEE Conference on Decision and Control (CDC) 2023*

# Quality of Service

# What is QoS?

- **Intuition** - Want each agent to be "sufficiently happy"
- Each agent $n$ has their own QoS requirement $\lambda_n$
- Local Objective: $\boldsymbol{u_n(x_1, \ldots, x_N) \geq \lambda_n}$

# Example: Power Control in Wireless Networks




- Players - Transmitters
- Action - Transmission Power
- Utility - Signal-to-Interference Ratio (SIR) or Throughput
- Vast literature on obtaining QoS for such games
  - Foschini et al. (1993), Yates (1995), Biguesh et al. (2011), etc.
  - Employ very specific techniques

# Tug-of-War Games

**Intuition**: Increase in player 1's action reduces rewards for all other players

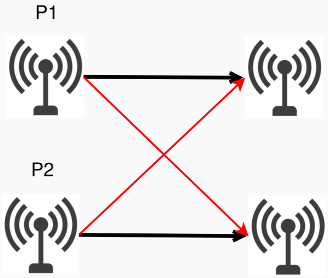
**Definition 1 (Tug-of-War Game)**

A game is a ToW game if the utility function is continuously differentiable and satisfies

$$\frac{\partial u_n(\mathbf{x})}{\partial x_m} < 0, \ \forall m \neq n.$$

Also $u_n(\mathbf{x}) = 0$ if $x_n = 0$ and $u_n(\mathbf{x}) \geq 0, \ \forall x$.

# Application 1: Power Control in Wireless Networks




- Players - Transmitters
- Action - Transmission Power
- Utility - Signal-to-Interference Ratio (SIR)

# Problem Formulation

- Action set $\mathcal{X}_n$ for each player: $\mathcal{X}_n \coloneqq [0, B_n] \subseteq \mathbb{R}$
- Each player chooses action $x_n(t)$ at each time $t \in \{0, 1, 2, \ldots\}$
- Observes noisy reward $y_n(t) = u_n(\mathbf{x}(t)) + M_t$
- Wish $\mathbf{x}(\mathbf{t}) \xrightarrow{a.s.} \hat{\mathbf{x}}$ where $u_n(\hat{\mathbf{x}}) \geq \lambda_n$ for all $n$

# Application 2: Activation in Sensor Networks


- Player: Sensors in a network
  - Collect data and also relay observations from other sensors
  - *On* (awake) or *Off* (asleep) at each time with some probability
  - When sensor is *off*: neither collects, nor relays
- Action $x_n$: sleeping probability for player $n$
- Utility for player $n$: $\alpha\pi_n(\mathbf{x}) - \beta B(x_n)$
  - $\pi_n(x)$: probability that player $n$'s packets reach their destination
    - Need all sensors in route to destination to be active for packet to reach destination
  - $B(x_n)$: battery usage of player $n$

# Application 2: Activation in Sensor Networks


- Action for player $n$: $x_n$ is sleeping probability for player $n$
- Utility for player $n$: $\alpha\pi_n(\mathbf{x}) - \beta B(x_n)$
  - $\pi_n(x)$: probability that player $n$'s packets reach their destination
  - $x_m \uparrow \implies \pi_n(\mathbf{x}) \downarrow \forall n$
  - $\boldsymbol{x_m \uparrow \implies u_n(\mathbf{x}) \downarrow \forall m \neq n}$
  - $x_m \uparrow \implies B(x_m) \downarrow$

# General Setting

Why can Power Control algorithms not work for general ToW games?

- Multiple Equilibria
- Boundary Issues
- Unknown System
- Handling Noise

# Goals

**Problem 1**

*Design a distributed algorithm which requires "little" communication between agents such that* $\mathbf{x}(t) \xrightarrow{a.s.} \hat{\mathbf{x}}$*, such that* $u_n(\hat{\mathbf{x}}) \geq \lambda_n$*, for all* $n$

**Subproblem 1**

$\mathbf{x}(t) \longrightarrow \mathbf{x}_*$*, where* $\mathbf{x}_*$ *is the minimal point s.t.* $u_n(\mathbf{x}_*) \geq \lambda_n$*, for all* $n$

# Tug-of-Peace

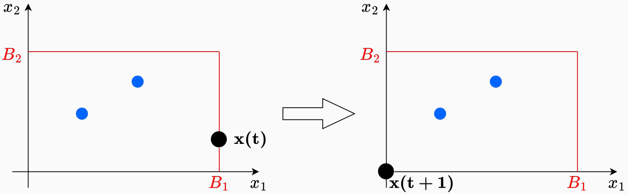
# Intuition - Iteration

**Iteration:**

$$x_n(t+1) = x_n(t) + \eta(t)(\lambda_n - u_n(\mathbf{x(t)})).$$

- Increase action if receive reward lower than QoS requirement
- Decreases rewards for other players
- Other players also increase their action
- ‘Cooperative’ increase in actions leads to convergence

# Intuition - Boundary Effects




**When at boundary:**

- Send *alarm* signal to every player.
- All players reset to action $0$ on receipt of *alarm* signal

**Intuition:**

- **1-bit signal** to avoid the possibility of being stuck at boundary
- Resets iteration

# Tug-of-Peace Algorithm

**Algorithm 1**

**Initialization:** Let $x_n(0) = 0, \ \forall n$.

**At timesteps** $t = 0, 1, \ldots,$ **each player** $n$

(1) Plays action $x_n(t)$ and observes a noisy reward $y_n(t)$.

(2) Updates their action as follows:

$$x_n(t+1) = x_n(t) + \eta(t)(\lambda_n - y_n(t)).$$

(3) Transmits signal $s_n = 1$ if $x_n(t+1) = B_n$, otherwise it does nothing (i.e., $s_n = 0$).

(4) Resets action to $0$, i.e., $x_n(t+1) = 0$ upon receiving $s_m = 1$ from some player $m$.

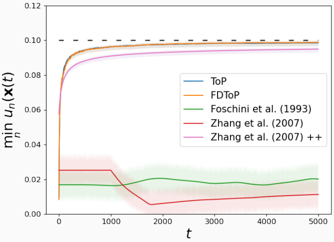
**End**

# Results

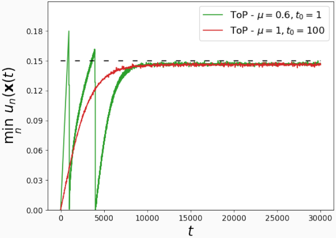
**Theorem 1**

1. *If the QoS requirements are feasible, then the iterates of the ToP algorithm a.s. converge to an equilibrium point* $\hat{\mathbf{x}}$ *such that* $u_n(\hat{\mathbf{x}}) \geq \lambda_n, \; \forall n.$
2. *The reset to* $\mathbf{x} = \mathbf{0}$ *happens only finitely often.*
3. *With high probability (depending on stepsize), the iterates converge to* $\mathbf{x}_*$*, where* $\mathbf{x}_*$ *is the minimal point which satifies the QoS requirements for all agents.*

# Numerical Results




**(a)** Power Control with $N = 50$ players




**(b)** Sensor Activation

# Proof Sketch

- **Stochastic Approximation**[3]: Iterates $\mathbf{x}(t)$ of ToP algorithm asymptotically track the solutions of the ODE

$$\dot{\mathbf{x}}(t) = \lambda - \mathbf{u}(\mathbf{x}(t))$$

- **Cooperative ODE**[4]: An ODE of form $\dot{\mathbf{x}}(t) = \mathbf{h}(\mathbf{x}(t))$, where

$$\frac{\partial h_n(\mathbf{x})}{\partial x_m} > 0$$

converges to a set of equilibria.

[3] Borkar (2022)
[4] Hirsch et al. (2003)

# Proof Sketch

- **Domain of Attraction**[5]: $\mathbf{x} = \mathbf{0}$ lies in the domain of attraction of the minimal equilibrium point $\mathbf{x}_*$ for the ODE: $\dot{\mathbf{x}}(t) = \lambda - \mathbf{u}(\mathbf{x}(t))$
  - For any point $\hat{\mathbf{x}}$ which satisfies $u_n(\hat{\mathbf{x}}) \geq \lambda_n$ for all $n$, $x_{*_n} \leq \hat{x}_n$ for all $n$.
- **Concentration:**[6] If initiated in the domain of attraction of $\mathbf{x}_*$, the iterates $\mathbf{x}(t)$ stay in a $\epsilon$-ball around $\mathbf{x}_*$ for all $t > T$ with high probability.

[5] Hirsch (1985)
[6] Thoppe et al. (2019)

# Summary

# Summary

- Learning desirable equilibria of unknown multi-agent systems
- Providing Quality of Service guarantees
    - Tug-of-War games
    - Tug-of-Peace algorithm
- Extensions for this work:
    - Asynchronous system
    - Finite-time guarantees

# Publications

## Multi-Agent Systems

1. S. Chandak, I. Bistritz, N. Bambos, "Tug of Peace: Distributed Learning for Quality of Service Guarantees", submitted to *IEEE Conference on Decision and Control (CDC) 2023*
2. S. Chandak, I. Bistritz, N. Bambos, "Equilibrium Bandits: Learning Optimal Equilibria of Unknown Dynamics", in *International Conference on Autonomous Agents and Multiagent Systems (AAMAS) 2023*

## Theoretical Reinforcement Learning

1. S. Chandak, V. S. Borkar and P. Dodhia, "Reinforcement Learning in Non-Markovian Environments", submitted to *Systems and Control Letters.*
2. S. Chandak, V. S. Borkar and H. Dolhare, "A Concentration Bound for LSPE($\lambda$)", in *Systems and Control Letters*, January 2023
3. S. Chandak, V. S. Borkar and P. Dodhia, "Concentration of Contractive Stochastic Approximation and Reinforcement Learning", in *Stochastic Systems*, July 2022

# Thank You!